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COURSE SYLLABUS

| General information | | | | |
|----------------------------|---|---|--|--|
| Course title | Algebra 2 | | | |
| Study programme | Graduate course Discrete Mathematics and Its Applications | | | |
| Year of study | 1st | | | |
| Course status | Mandatory | | | |
| Course homepage | merlin.srce.hr | | | |
| Language of instruction | English | | | |
| Credit values and modes of | ECTS credits / student workload | 6 | | |
| instruction | Hours (L+E+S) | 30 + 30 + 0 | | |
| | Name and surname | Vera Tonić | | |
| | Office | O-303 | | |
| Lecturer | Office hours | Wednesday, 14:00-15:00 Thursday, 14:00-15:00 | | |
| | Phone number | 584-684 | | |
| | E-mail | vera.tonic@math.uniri.hr | | |
| | Name and surname | | | |
| | Office | | | |
| Teaching assistant | Office hours | | | |
| | Phone number | | | |
| | E-mail | | | |

1. COURSE DESCRIPTION

1.1. Course objectives

The main course objective is to get students acquainted with:

- basic notions of ring theory, especially polynomial rings;
- basic notions of field theory and field extensions;
- basic notions of Galois theory.

1.2. Course prerequisites

Students are required to have taken Algebra 1 and to have basic knowledge of algebraic structures.

1.3. Learning outcomes

After completing the course, students are expected to:

- be able to define, give examples, and recognize basic algebraic structures with two operations (A7, B7);
- understand and be able to apply correctly the notions of ring, ideal, and ring homomorphism (A7,B7);
- know and be able to prove basic theorems related to polynomial rings (F3, B7);
- know and be able to apply correctly different types of field extensions (A7,B7,C7);
- know basic theorems and notions of Galois theory (A7, B7);
- be able to solve, with appropriate explanation, problems on determining the Galois group of a field extension (A7, B7).

1.4. Course content



Rings and ideals. Integral domains, Euclidean domains, principal ideal domains, unique factorization domains. Polynomial rings. Field extensions (simple, algebraic, of finite degree, normal, separable, radical). Field automorphisms and Galois groups, Galois field extensions and the fundamental theorem of Galois theory. Splitting fields for polynomials and algebraic closure. Solvability of Galois group as solvability of the appropriate equation by radicals. Finite fields.

| 1.5. Modes of instruction | ➢ lectures ➢ seminars and workshops ➢ exercises ➢ e-learning ☐ field work | independent work multimedia and the internet laboratory tutorials mentoring work consultative teaching other |
|------------------------------|---|--|
| 1.6. Comments | | |
| | | |

1.7. Student requirements

Students are expected to attend classes and actively participate in them, and to take notes. They are required to achieve a certain number of points during the semester and to pass the final exam. Details on the grading system are explained in the following paragraph, titled Grading policy.

2. GRADING POLICY

2.1. Grading of students' work during the semester and on the final exam

Students' work will be evaluated and assessed during the semester and on the final exam. The total number of points a student can achieve during the semester is 70. To gain access to the final exam, students are required to earn at least 50% of the total points during the semester, that is, 35 out of 70 points. Moreover, in order to gain access to the final exam, students have to fulfill the minimal requirements described in the table below. The maximal number of points that can be achieved on the final exam is 30, with the passing score being 50%, that is, 15 out of 30 points. The final exam is an oral exam, and students should be prepared for questions of theoretical nature, as well as questions on problem solving.

Students who achieve between 0% and 49.9%, through all of the activities evaluated, will receive the grade F (Insufficient), they cannot get the ECTS credits and will have to enroll in the course again. The same is true for students who do not pass the final exam within the three attmepts offered during the examination period.

METHODS OF EVALUATION AND ASSESMENT OF STUDENTS' WORK DURING SEMESTER

MIDTERM EXAMS (60 points total):

During the semester, **two 90-minutes long written exams** will be conducted, one approximately in the middle of the semester, and one near the end of the semester (we shall refer to both of these exams as "midterms"). On each of these exams a student can achieve at most 30 points. If the necessity arises, there will be one make-up midterm, in the very end of the semester (exact time tba), on which a student can attempt to achieve a better score, replacing a bad score on only one of the above mentioned midterms. Note, however, that if a student decides to use the make-up midterm exam, the grade on the make-up exam will be considered as the final one for that exam, regardless whether it is better or worse than the grade the student was trying to repair.

QUIZZES (very short tests) (10 points total):

Once a week, there will be a homework problem set assigned, that students should work on, but they need not submit. Instead, during the semester, there will be **five short quizzes during class**, i.e., short written tests, up to 20 minutes in duration, usually containing a short theoretical question (on basic definitions or theorems we studied in class), and one problem to solve, either taken directly from the previously assigned homework, or very similar to a problem from the previously assigned homework. Each student can earn at most 10 points for their quizzes in total. **There are no make-ups for missed or low-scoring quizzes** (even if you miss a quiz for a very legitimate reason), **but there are also no minimal requirements for quizzes**, that is, if a student misses all of the quizzes, this student can still qualify for access to final exam if they achieve at least 35 points in total on their two midterm exams.



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| 2.2. Minimal requirements for access to the final exam / passing grade | | | |
|--|--|--|--|
| ACTIVITY | MINIMAL NUMBER OF POINTS REQUIRED | | |
| Midterm exam 1 | at least 15 out of 30 points (50%) | | |
| Midterm exam 2 | at least 15 out of 30 points (50%) | | |
| Quizzes | (maximal score of 10 points) there is no minimal requirement, but the points you do not earn on quizzes (up to a total passing score of 35) must be earned on the two midterm exams | | |
| TOTAL: | 35 out of 70 points (50%) | | |
| OTHER REQUIREMENTS: | | | |
| 2.3. Final grade – grading scale | | | |
| According to the total number of points achieved during the semester and on the final exam, the final grade is | | | |

determined using the following scale: GRADE POINTS Excellent (5), A 90% - 100% Very good (4), B 75% - 89,9% Good (3), C 60% - 74,9% Sufficient (2), D 50% - 59,9% Insufficient (1), F 0% - 49,9%

3. LITERATURE

3.1. Required literature

T.W. Hungerford: Algebra, Reinhart and Winston, NY, 1989.

3.2. Recommended literature

I. Stewart: Galois Theory, Chapmann and Hall, London, 1973.

4. ADDITIONAL INFORMATION

4.1. Class attendance

Taking midterm exams is mandatory. Class attendance is recommended, though not mandatory. Students are required to inform themselves about all classes that they missed. No disturbance of classes will be tolerated, and the usage of calculators, cell phones or electronic devices of any kind is absolutely forbidden during class.

4.2. Informing students

All relevant course information will be posted on the course homepage, on **merlin.srce.hr**. It is the student's personal responsibility to be regularly informed, by checking the course web page and their official email.

4.3. Other relevant information

Students are expected to have a high degree of independence and responsibility in their work. During the course, an active aproach to learning will be encouraged.

While performing the tasks specified within the course curriculum, students must represent their own work, and they are not allowed to use someone else's text as their own. Submitting work that incorporates someone else's ideas without citation is considered to be stealing intellectual property, and can be punished according to the university regulations. Any work required to be submitted for grading should be prepared according to the instructions given during class or on class web page.



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4.4. Assessment of quality and performance for the course

Quality of the lectures is assessed in accordance with the regulations of the Department of Mathematics and the University of Rijeka. At the end of the semester, an anonymous survey will be conducted, in which students will evaluate the quality of the lectures. Additionally, the department will conduct the analysis of the examination results.

4.5. Examination period

| Final exam (1st examination period) | 23 rd June 2021, 10 a.m. |
|-------------------------------------|---|
| Final exam (2nd examination period) | 7 th July 2021, 10 a.m. |
| Final exam (3rd examination period) | 1 st September 2021, 10 a.m. |

5. COURSE OUTLINE*

| DATE | TIME | MODE OF INSTRUCTION | ΤΟΡΙϹ | GROUP | LECTURE HALL |
|------------|-------------|------------------------|--|-------|-----------------|
| 3.3.2021. | 12:15-13:45 | L/E | Review of basic definitions: ring, integral domain, division ring, field | | 360 |
| 4.3.2021. | 12:15-13:45 | L/E | Examples of rings and fields | | 356 |
| 10.3.2021. | 12:15-13:45 | L/E | Ring homomorphisms, subrings, ideals | | 360 |
| 11.3.2021. | 12:15-13:45 | L/E | Ideals, quotient ring, prime ideals | | 356 |
| 17.3.2021. | 12:15-13:45 | L/E | Factorization in commutative rings, principal ideal domain, Euclidean domain | | 360 |
| 18.3.2021. | 12:15-13:45 | L/E | Quiz 1 Euclidean domain and norm, unique factorization domain | | 356 |
| 24.3.2021. | 12:15-13:45 | L/E | Ring of polynomials | | 360 |
| 25.3.2021. | 12:15-13:45 | L/E | Polynomial ring, division algorithm in polynomial rings | | 356 |
| 31.3.2021. | 12:15-13:45 | L/E | Factorization in polynomial rings | | 360 |
| 1.4.2021. | 12:15-13:45 | L/E | Quiz 2 Fields of fractions | | 356 |
| 7.4.2021. | 12:15-13:45 | L/E | Irreducibility criteria for polynomials | | 360 |
| 8.4.2021. | 12:15-13:45 | L/E | Cyclotomic polynomials | | 356 |
| 14.4.2021. | 12:15-13:45 | L/E | Field extensions, simple field extensions | | 360 |
| 15.4.2021. | 12:15-13:45 | | FIRST MIDTERM EXAM | | 356 |
| 21.4.2021. | 12:15-13:45 | L/E | Field extensions, minimal polynomial | | 360 |
| 22.4.2021. | 12:15-13:45 | L/E | Algebraic and transcendental extensions | | 356 |
| 28.4.2021. | 12:15-13:45 | L/E | Algebraic field extensions | | 360 |
| 29.4.2021. | 12:15-13:45 | L/E | Quiz 3 Algebraic closure | | 356 |
| 5.5.2021. | 12:15-13:45 | L/E | Field isomorphisms and automorphisms, Galois group | | 360 |



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| 6.5.2021. | 12:15-13:45 | L/E | Field automorphisms, Galois group | 356 |
|------------|-------------|-----|---|-----|
| 12.5.2021. | 12:15-13:45 | L/E | Galois field extensions, Fundamental theorem of Galois theory | 360 |
| 13.5.2021. | 12:15-13:45 | L/E | Quiz 4 Fundamental theorem of Galois theory | 356 |
| 19.5.2021. | 12:15-13:45 | L/E | Splitting fields, algebraic closure | 360 |
| 20.5.2021. | 12:15-13:45 | L/E | Splitting fields, algebraic closure | 356 |
| 26.5.2021. | 12:15-13:45 | L/E | Separable and normal field extensions | 360 |
| 27.5.2021. | 12:15-13:45 | L/E | Quiz 5 Galois group of a polynomial, discriminant | 356 |
| 2.6.2021. | 12:15-13:45 | L/E | Finite fields | 360 |
| 3.6.2021. | | | Holiday, no class | |
| 9.6.2021. | 12:15-13:45 | L/E | Finite fields, continued | 360 |
| 10.6.2021. | 12:15-13:45 | | SECOND MIDTERM | 356 |

*Minor changes are possible. Up to 40% of lectures can be held online.

L – lectures

E - exercises (tutorials)